

## Exam III

April 16, 2015

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Place an  $\times$  through your answer to each problem.

- |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|
| 1.  | (a) | (b) | (c) | (d) | (e) |
| 2.  | (a) | (b) | (c) | (d) | (e) |
| 3.  | (a) | (b) | (c) | (d) | (e) |
| 4.  | (a) | (b) | (c) | (d) | (e) |
| 5.  | (a) | (b) | (c) | (d) | (e) |
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MC. \_\_\_\_\_  
11. \_\_\_\_\_  
12. \_\_\_\_\_  
13. \_\_\_\_\_  
14. \_\_\_\_\_  
15. \_\_\_\_\_  
Tot. \_\_\_\_\_

**Multiple Choice**

1. (5 pts.) There are five lighthouses in the state of Alabama. Their heights (in feet) are  
45, 63, 41, 49, 132

The mean height is 66 feet (you don't have to verify this). What is the (population) variance for the heights of lighthouses in Alabama?

- (a) 1430                                      (b) 37.82                                      (c) 1144  
(d) 33.82                                      (e) 66

2. (5 pts.) The scores for an exam worth 100 points have mean 80 and standard deviation 10. If the scores are normally distributed, what percentage of students got an A (had a score greater than 90) on the exam?

- (a) 15.87%                                      (b) 0%    (c) 50%  
(d) 34.13%                                      (e) 84.13%

3. (5 pts.) Adam rolls a 4-sided die 8 times. What is the probability that he rolls at least 6 fours?

- (a)  $C(8, 6)(0.25)^6(0.75)^2$
- (b)  $C(8, 6)(0.25)^6(0.75)^2 \cdot C(8, 7)(0.25)^7(0.75)^1 \cdot C(8, 8)(0.25)^8(0.25)^0$
- (c)  $C(8, 6)(0.75)^6(0.25)^2 + C(8, 7)(0.75)^7(0.25)^1 + C(8, 8)(0.75)^8(0.25)^0$
- (d)  $C(8, 6)(0.75)^6(0.25)^2$
- (e)  $C(8, 6)(0.25)^6(0.75)^2 + C(8, 7)(0.25)^7(0.75)^1 + C(8, 8)(0.25)^8(0.25)^0$

4. (5 pts.) Let  $Z$  be a standard normal random variable. Find the number  $x$  so that

$$P(-1 \leq Z \leq x) = 0.4967.$$

- (a) 0.1587
- (b) 0.4
- (c) 0.4967
- (d) 0.6554
- (e) 1

5. (5 pts.) The following is the probability distribution for a random variable,  $X$ ,

k	P(X=k)
0	0.3
1	0.5
2	0.1
3	0.1

What is  $\sigma(X)$  (rounded to 3 decimal places)?

- (a) 0.800                      (b) 0.447                      (c) 0.894  
(d) 1.225                      (e) 1.500

6. (5 pts.)

Consider the system of inequalities

$$\begin{aligned}x + y &\leq 5 \\x - 2y &\leq 2 \\x &\geq 1\end{aligned}$$

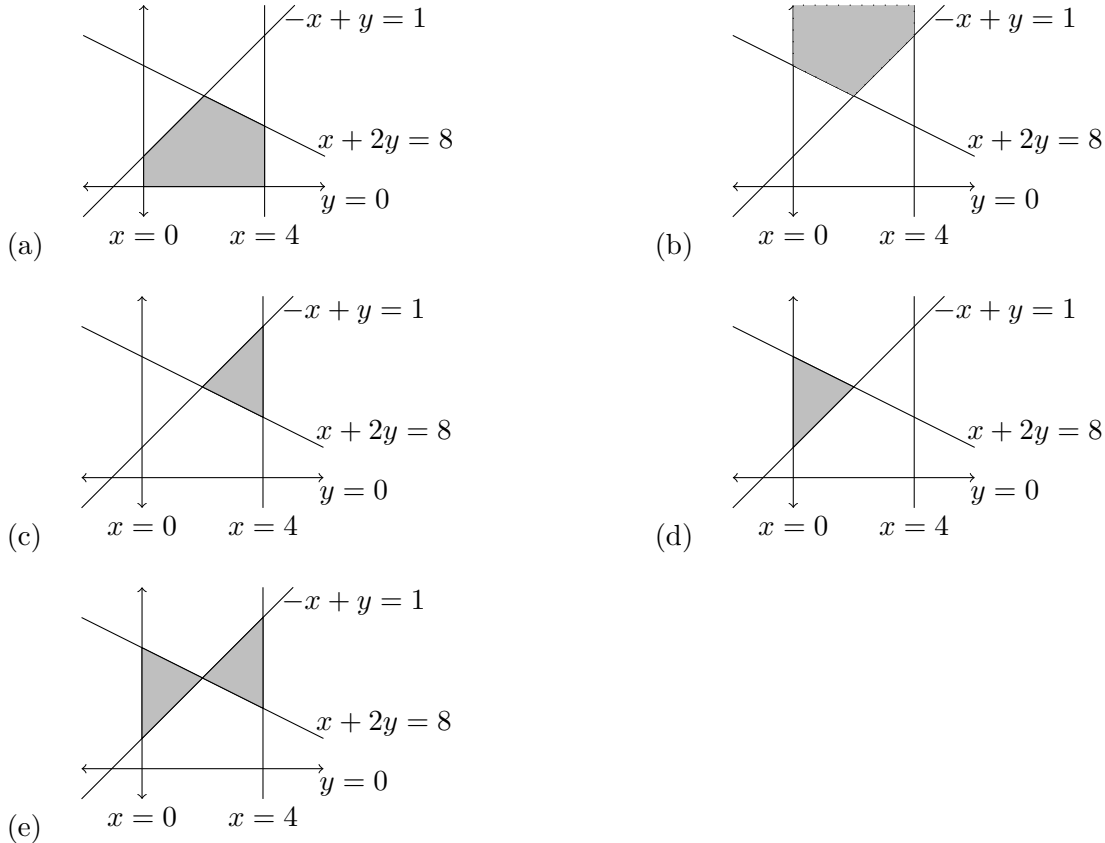
Which of the following points is in the feasible set defined by these inequalities? (Notice that we are not asking you to graph the feasible set.)

- (a) (3, 3)                      (b) (0, 4)                      (c) (5, 1)  
(d) (2, -1)                      (e) (1, 2)

7. (5 pts.) Consider the system of inequalities

$$\begin{aligned} -x + y &\geq 1 \\ x + 2y &\leq 8 \\ x &\leq 4 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

Which of the following shaded regions is the feasible set for this system of linear inequalities?



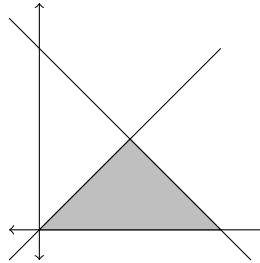
8. (5 pts.) Bob and Bill's Glassworks is a store that makes and sells glass lamps and crystal balls. Bob makes the items and Bill puts in the finishing touches. Each glass lamp takes 2 hours for Bob to make and 1 hour for Bill to put finishing touches on. Each crystal ball takes 4 hours for Bob to make and 2 hours for Bill to put finishing touches on. Bob works 50 hours every week and Bill works 40 hours every week. Let  $x$  be the number of glass lamps made and  $y$  the number of crystal balls made. Which of the following is one of the inequalities that come from this information? (**Pay attention to  $\leq$  versus  $\geq$ .** There is no objective function in this problem.)

- (a)  $x + 2y \leq 40$                       (b)  $x + 2y \geq 40$                       (c)  $2x + y \leq 40$   
 (d)  $2x + y \geq 40$                       (e)  $2x + 4y \geq 50$

9. (5 pts.)

Consider the system of inequalities whose feasible set is sketched to the right.

$$\begin{aligned} -x + y &\leq 0 \\ x + y &\leq 6 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$



What is the maximum value of the objective function  $2x + 3y$  subject to the above constraints?

- (a) 0                      (b) 12                      (c) 18                      (d) 15  
 (e) There is no maximum value with these constraints and objective function.

10. (5 pts.) Find the following matrix product:

$$\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

- (a)  $\begin{bmatrix} 1 & 1 \end{bmatrix}$                       (b)  $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$                       (c)  $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$   
 (d)  $\begin{bmatrix} 4 & -1 \end{bmatrix}$                       (e) This product is impossible

**Partial Credit**

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

**11.** (10 pts.) Sunny brand orange juice is produced in 40 oz bottles. However, the machines that dispense the orange juice into the bottles are not able to dispense exactly 40 oz of orange juice each time. The amount of juice dispensed is normally distributed with mean 40.5 (ounces) and standard deviation 5 (ounces).

(a) What percentage of bottles of Sunny brand orange juice produced contain less than the advertised amount of juice (less than 40 ounces)?

(b) What percentage of bottles of Sunny brand orange juice produced contain between 39 and 41 ounces of juice?

**12.** (10 pts.) My dog loves to play fetch. I know from past experience that he catches the ball in the air 80 percent of the time (so he misses 20 percent of the time). Suppose I throw the ball 10 times. Let  $X$  be the number of times my dog catches the ball. You do not need to simplify your answers in parts (a) or (b).

(a) What is the probability that my dog catches the ball exactly 7 times?

(b) What is the probability that my dog catches the ball between 4 and 7 times, inclusive? (That is, compute  $P(4 \leq X \leq 7)$ .)

(c) Sketch the normal curve that best approximates the probability distribution of the random variable  $X$ . Be sure to include labeled axes and a scale. Also, give the mean and standard deviation of the normal curve you're drawing. (Hint: The mean and standard deviation should be the mean and standard deviation for  $X$ . You should simplify your answer here.)



**13.** (10 pts.) Tom pays 5 dollars to play the following game. First, he rolls a 4-sided die. The number he rolls gives the number of dollars he wins. If Tom rolls a 2, 3, or 4, the game is over. If he rolls a 1, he gets to flip a coin. If the coin flip results in heads, he gets 10 dollars more, and the game ends. If the coin flip results in tails, he wins no additional money, and the game ends. Let  $X$  be Tom's net earnings.

- (a) Give a probability distribution for  $X$ . (A tree diagram might be helpful for determining the probabilities.)

$k$	$P(X=k)$

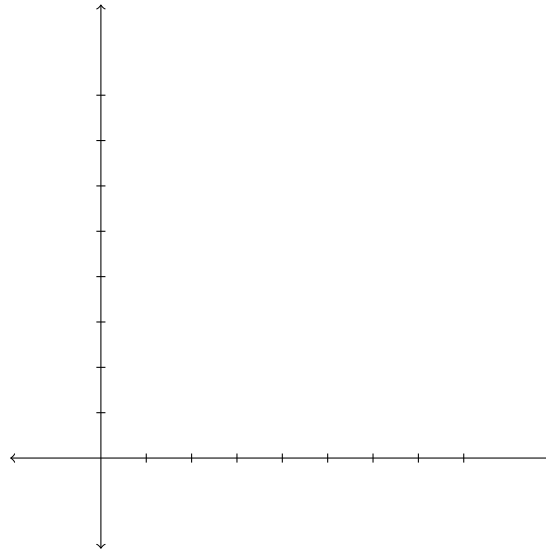
- (b) What is  $E(X)$ ?

- (c) If Tom plays the game 100 times, should he expect to win money or lose money? How much should he expect to win or lose? Explain your answer.

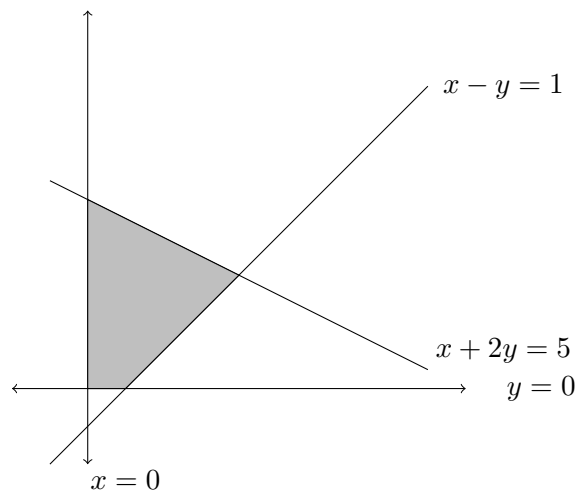
14. (10 pts.)

- (a) Graph the feasible set of the region given by the given inequalities using the axes provided. Be sure to shade in the entire feasible set (and nothing else). In this part of the problem we are not asking you to label the corners of the feasible set, but please label your lines.

$$\begin{aligned} 2x - y &\geq -1 \\ x + y &\leq 7 \\ x &\geq 0 \\ y &\leq 3 \\ y &\geq 0 \end{aligned}$$



- (b) Find the coordinates of the corners of the following feasible set. You can put your answers in the figure itself.





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